1961

Moving target effects upon matched filter correlators.

James, Joe Mitchell

Monterey, California: U.S. Naval Postgraduate School

http://hdl.handle.net/10945/12695

MOVING TARGET EFFECTS UPON MATCHED FILTER CORRELATORS

JOE MITCHELL JAMES

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Joe Mitchell James

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MATCHED FILTER CORRELATORS

by

Joe Mitchell James

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
Monterey, California

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This work is accepted as fulfilling

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IN
ENGINEERING ELECTRONICS

from the

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ABSTRACT

Preface

This paper considers the effect of a moving target upon the output of a detection system employing matched filters. A system employing a pair of pulse compression signals is then proposed which would present accurate target range and a measure of target radial velocity.

The writer wishes to thank Associate Professor George Hahn and Professor Lawrence E. Kinsler of the United States Naval Postgraduate School for their assistance, encouragement and cooperation in the preparation of this paper.

TABLE OF CONTENTS

GLOSSARY OF NOTATION

u = target velocity

C = velocity of propagation

r = target range

R_o = target range at time zero

Δt_p = one half path time

t_e = receiver time

t = time

f = frequency cycles/sec

ω = frequency radians/sec

$\delta(t)$ = the delta function

$u(t)$ = the step function

$f(t)$ = a function of time

$h(t)$ = impulse response of filter

$g(t)$ = response of filter to function

$\mathcal{L}^{-1}$ = inverse Laplace transform

$f^D(t)$ = a doppplered function

$f^{\tilde{D}}(t)$ = approximation of a dopplered function

τ_o = delay due to target range

τ = increments of time

* = complex conjugate (superscript)

$*$ = convolution

$Z(u)$ = complex Fresnel Integral

INTRODUCTION

The study of doppler effect in detection theory usually assumes that the frequency components of a signal have been shifted by a uniform amount, which is a function of target and propagation velocity and the mean frequency of the signal. Much analysis and some system design, such as the doppler radar, has been done on the basis of the above assumption.

The supposition that doppler is a shift of frequency by a constant amount is an approximation, but a fair one in many cases. Particularly is it good when referring to electromagnetic propagation where the velocity of propagation is 3×10^8 meters per second is large compared to the velocity of the target which may be no more than 3×10^2 meters per second (mach one). Also, and most important, the bandwidth is nearly always a very small part of the "carrier frequency", e.g., a 1 kmc. pulse radar whose pulse width is one microsecond has a bandwidth one one-thousandth of the carrier frequency. Conditions are not so favorable for acoustic signals propagating in water. The velocity of propagation is about 1.5×10^3 meters per second and the velocity of the receiver or transmitter may be 1.5×10 meters per second, (30 knots). However, the carrier frequency to bandwidth ratio is usually large, e.g. a 5 kcs. pulsed sonar whose pulse width is two seconds has a bandwidth one ten-thousandth of the carrier frequency.

The nature of water as an acoustic medium has encouraged the use of frequencies as low as possible, compatable with the decrease in target strength which occurs with lower frequencies. In order to obtain accurate range the bandwidth of the signal has been increased. Some systems such as the "pulse compression" system, or "chirp", utilize bandwidth to achieve processing gain as well as accurate range. It is these conditions, where target velocity is comparable to propagation velocity and bandwidth

is nearly equal to the mean frequency of the signal, which have stimulated a more exact study of the doppler effect.

In order to make a study of the effect of doppler upon a system, it is necessary to first find what is the effect of a moving target upon a received signal.

Consider the situation shown in figure I.

c = propagation velocity

u = target volocity

r = target range, a function of time.

$R_o = r\ (o)$

From the figure it may be seen that:

$$r(t) = R_o - ut$$
$$c\Delta t_2 = r(t) - u\Delta t_2$$

And by simple algebra:

$$(c+u)\Delta t_2 = r(t) - ut$$

Defining Δt_r as the time for the echo to return to the target, it can be shown from the geometry of the figure that:

$$\Delta t_r = \frac{r(t) - u\Delta t_2}{c} = \frac{(c+u)\Delta t_2 - u\Delta t_2}{c} = \Delta t_2$$

Two new symbols are introduced and defined:

$\Delta t_p \equiv \Delta t_r = \Delta t_2 \equiv$ one half path time.

$t_e \equiv t + 2\Delta t_p \equiv$ time of occurrence of any event at the receiver.

$$t_e = t + 2\Delta t_p = t + \frac{2(R_o - ut)}{(c+u)}$$
$$= \frac{t(c-u)}{(c+u)} + \frac{2R_o}{(c+u)}$$
$$t = \frac{t_e(c+u)}{(c-u)} - \frac{2R_o}{(c-u)}$$

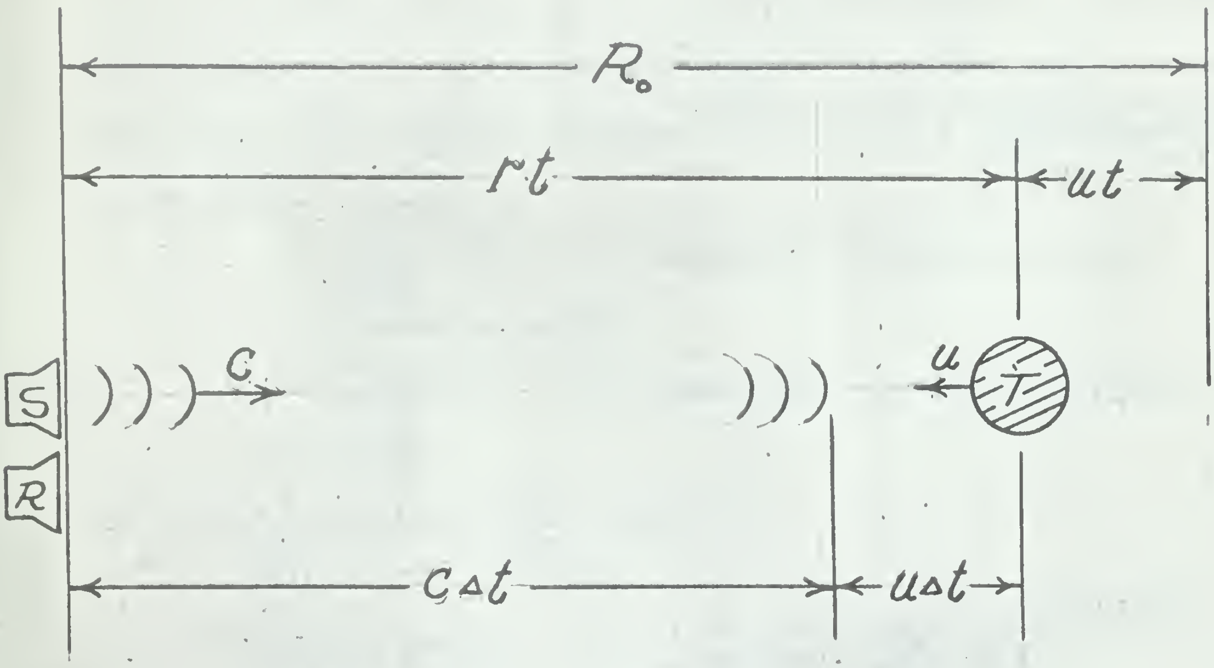


FIGURE 1

$$t = \left(t_e - \frac{2R_o}{C+U}\right) \times \frac{(C+U)}{(C-U)}$$

Therefore given any transmitted function $f(t)$:

$$f(t) = f\left(\left[t_e \frac{-2R_o}{C+U}\right]\left[\frac{C+U}{C-U}\right]\right)$$

where t_e is defined as the time of occurrence of any event at the receiver and therefore is time as kept by the receiver. It is seen that a transmitted function $f(t)$ dopplered, becomes $f^D(t) = f\left(\left[t - \frac{2R_o}{C+U}\right]\left[\frac{C+U}{C-U}\right]\right)$ at the receiver. Where;

$\frac{2R_o}{C+U} = \tau_o$ is a delay due to transmission time through the medium.

$\frac{C+U}{C-U} = a$ is a compression factor due to doppler effect.

The fourier transform of such a signal is:

$$f(t) \longrightarrow F(\omega)$$

$$f([t-\tau_o]a) \longrightarrow \frac{1}{a} F\left(\frac{\omega}{a}\right) e^{-j\omega\tau_o}$$

The approximation which is commonly used requires $\frac{1}{a} \cong 1$ and $\omega = \omega_o + \omega'$ where $\omega_o >> \omega'$

Such that:

$$\frac{1}{a}(\omega) = \frac{1}{a}(\omega_o + \omega') \cong \frac{1}{a}(\omega_o) + \omega'$$

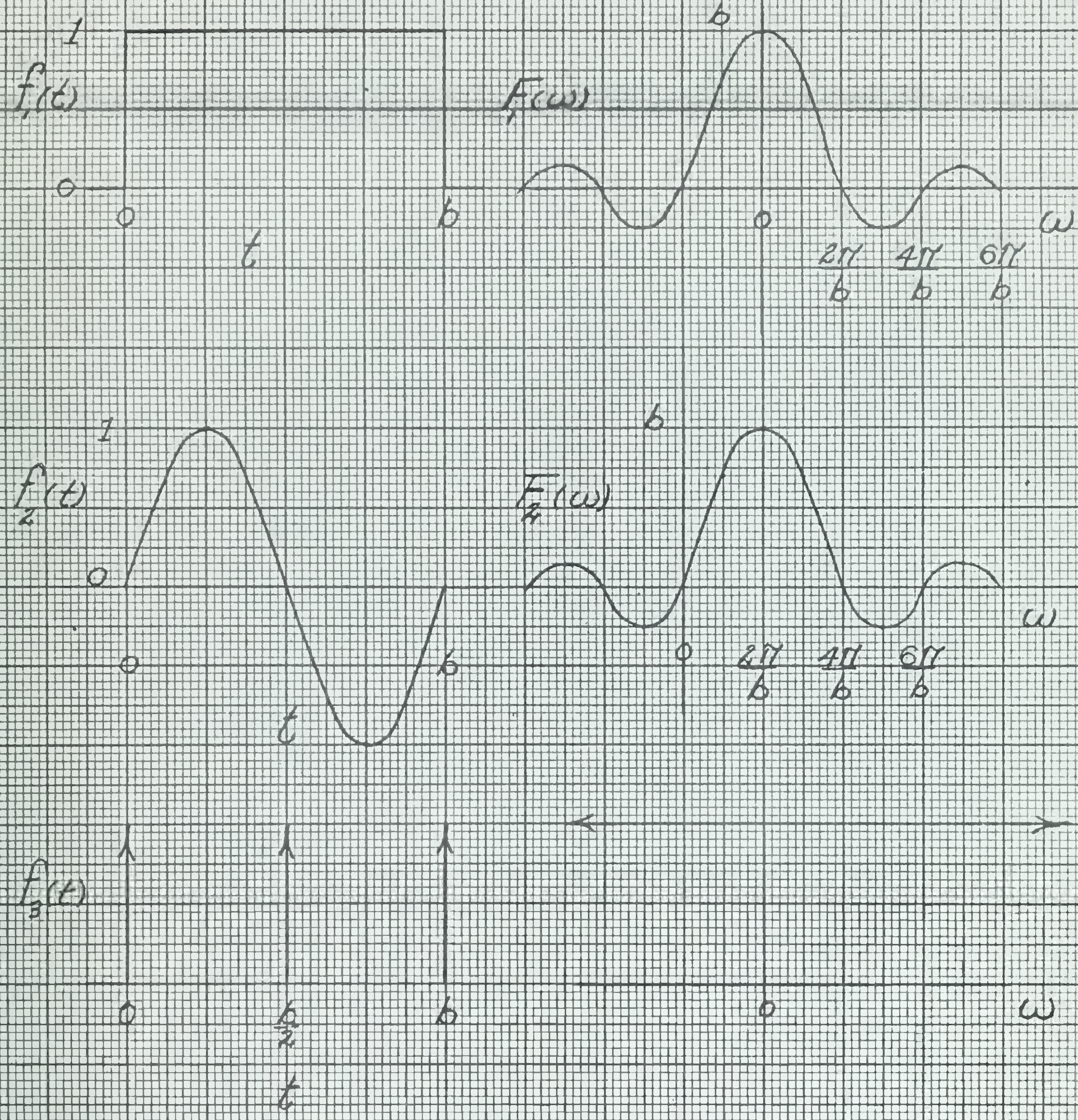
$$= (\omega_o + \omega') + \frac{(1-a)}{a}\omega_o$$

$$= \omega - \frac{2U}{C+U}\omega_o = \omega - \Delta\omega_o$$

Therefore if the origianl spectrum $F(\omega)$ was about ω_o, the spectrum as a result of doppler $F\left(\frac{\omega}{a}\right) \cong F(\omega - \Delta\omega_o)$ is about $\omega_o + \Delta\omega_o$

i.e. the spectral density has been displaced a constant amount $\Delta\omega_o$.

If ω_c is not much greater than ω' this approximation is not valid. It may be expected that this will be frequently the case in modern day accoustic modulation schemes.



1
$f_1(t)$
0
0
b
t
b
$F_1(\omega)$
0
ω
$\frac{2\pi}{b}$
$\frac{4\pi}{b}$
$\frac{6\pi}{b}$
1
$f_2(t)$
0
0
b
t
b
$F_2(\omega)$
ω
0
$\frac{2\pi}{b}$
$\frac{4\pi}{b}$
$\frac{6\pi}{b}$
$f_3(t)$
0
$\frac{b}{2}$
b
t
0
ω

FIGURE 2

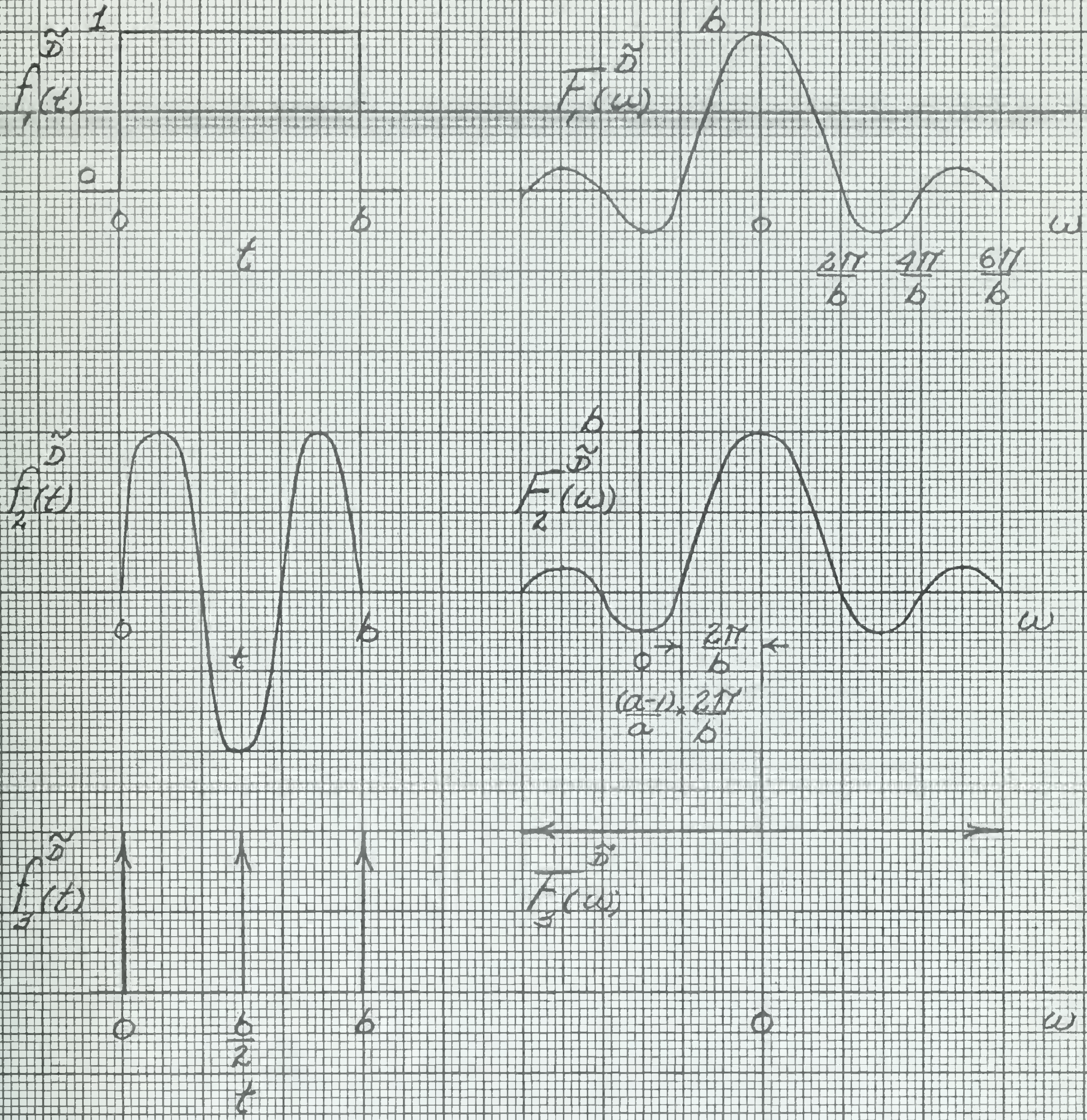

1
$\tilde{D}$
$f_1(t)$
0
0
b
t
b
$\tilde{D}$
$F_1(\omega)$
0
ω
$\frac{2\pi}{b}$
$\frac{4\pi}{b}$
$\frac{6\pi}{b}$
$\tilde{D}$
$f_2(t)$
0
b
t
b
$\tilde{D}$
$F_2(\omega)$
ω
0
$\frac{2\pi}{b}$
$\frac{(a-1)}{a} \times \frac{2\pi}{b}$
$\tilde{D}$
$f_3(t)$
0
$\frac{b}{2}$
b
t
$\tilde{D}$
$F_3(\omega)$
0
ω

FIGURE 3

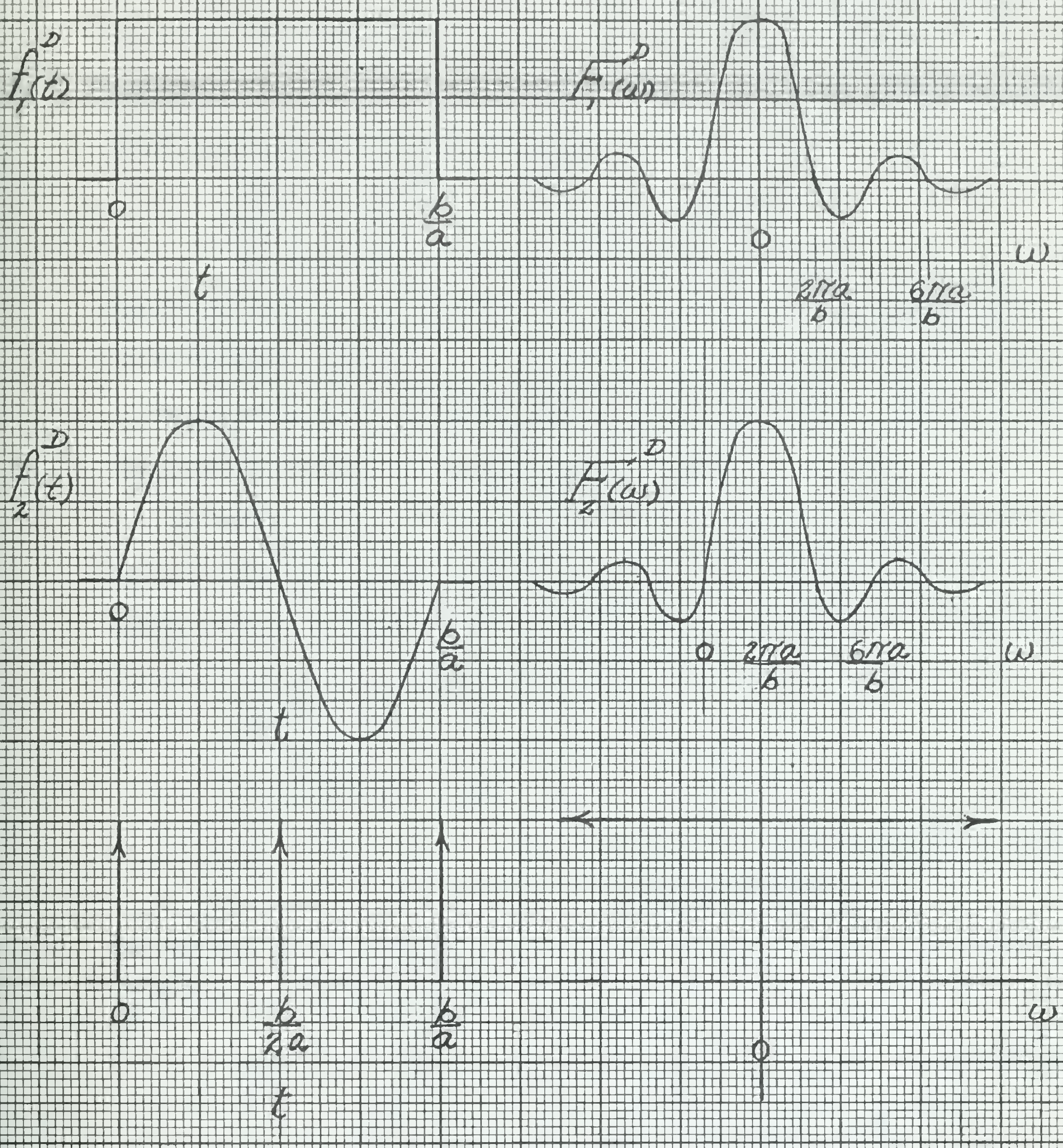

$f_1^D(t)$
0
$\frac{b}{a}$
t
$F_1^D(\omega)$
0
ω
$\frac{2\pi a}{b}$
$\frac{6\pi a}{b}$
$f_2^D(t)$
0
$\frac{b}{a}$
t
$F_2^D(\omega)$
0
$\frac{2\pi a}{b}$
$\frac{6\pi a}{b}$
ω
0
$\frac{b}{2a}$
$\frac{b}{a}$
t
ω
0

FIGURE 4

II

Before progressing to more complex wave forms, it is well to consider some familar and simple wave forms to note what doppler effects occur and how these differ from results concluded by assuming, as is frequently done, that the spectrum has been shifted by a uniform amount. Three simple wave forms are selected: $f_1(t) = u(t) - u(t-b)$

$$f_2(t) = [u(t) - u(t-b)] \, SIN(\tfrac{2\pi}{b})t$$

$$f_3(t) = \delta(t) + \delta(t - \tfrac{b}{2}) + \delta(t-b)$$

These functions are shown in Figure 2. The customary approximation of these dopplered functions yields functions shown in figure 3 (the effect of target range is neglected as being only a time delay). In actual fact the dopplered signals are as appear in Figure 4.

Of more interest is the matched filter response to these functions, as most receivers employ a matched filter or some approximation to a matched filter in order to maximize the signal to noise ratio. The matched filter is assumed to be matched to the undopplered signal, i. e.

$$h(t) = f^*(-t)$$

$$\mathcal{L}\, h(t) = H(s)$$

where $h(t)$ is the impulse response of the matched filter.

The matched filter response to these functions is computed below where $g(t)$ is the response in the time domain.

$$g_1(t) = h_1(t) * f_1(t) = \mathcal{L}^{-1} G(s) = \mathcal{L}^{-1} F(s) H(s)$$

$$= \mathcal{L}^{-1} \left(\frac{1}{s} - \frac{e^{-bs}}{s}\right)\left(\frac{1}{s} - \frac{e^{-bs}}{s}\right)$$

$$= \mathcal{L}^{-1} \left(\frac{1}{s^2} \quad \frac{2e^{-bs}}{s^2} \quad \frac{e^{-2bs}}{s^2}\right)$$

$$= t - 2(t-b) + (t-2b)$$

$$g_2(t) = h_2(t) * f_2(t) = \mathcal{L}^{-1} G_2(s) = \mathcal{L}^{-1} F_2(s) H_2(s)$$

$$= \mathcal{L}^{-1} \left(\frac{1}{s^2 + \left(\frac{2\pi}{b}\right)^2} - \frac{e^{-bs}}{s^2 + \left(\frac{2\pi}{b}\right)^2} \right)^2$$

$$= \mathcal{L}^{-1} \left(\frac{-1}{\left\{s^2 + \left(\frac{2\pi}{b}\right)^2\right\}^2} + \frac{e^{-bs}}{\left\{s^2 + \left(\frac{2\pi}{b}\right)^2\right\}^2} - \frac{1}{\left\{s^2 + \left(\frac{2\pi}{b}\right)^2\right\}^2} \right)$$

$$g_2(t) = \frac{-\sin\left(\frac{2\pi}{b}\right)t + \left(\frac{2\pi}{b}\right) t \cos\left(\frac{2\pi}{b}\right)t}{2\left(\frac{2\pi}{b}\right)} \qquad 0 < t < b$$

$$= \frac{\sin\left(\frac{2\pi}{b}\right)t - \left(\frac{2\pi}{b}\right)(t-2b)\cos\left(\frac{2\pi}{b}\right)t}{2\left(\frac{2\pi}{b}\right)} \qquad b < t < 2b$$

$$= 0 \qquad 2b < t$$

$$g_3(t) = h_3(t) * f_3(t) = \mathcal{L}^{-1} G_3(s) = \mathcal{L}^{-1} F_3(s) H_3(s)$$

$$= \mathcal{L}^{-1} \left(1 + e^{-\frac{b}{2}s} + e^{-bs}\right)^2$$

$$= \mathcal{L}^{-1} \left(1 + 2e^{-\frac{b}{2}s} + 3e^{-bs} + 2e^{-\frac{3b}{2}s} + e^{-2bs}\right)$$

$$= \delta(t) + 2\delta\left(t - \frac{b}{2}\right) + 3\delta(t-b)$$

$$+ 2\delta\left(t - \frac{3b}{2}\right) + \delta(t-2b)$$

These outputs are shown in Figure 5.

Computing the exact output of the matched filter due to a dopplered input yields:

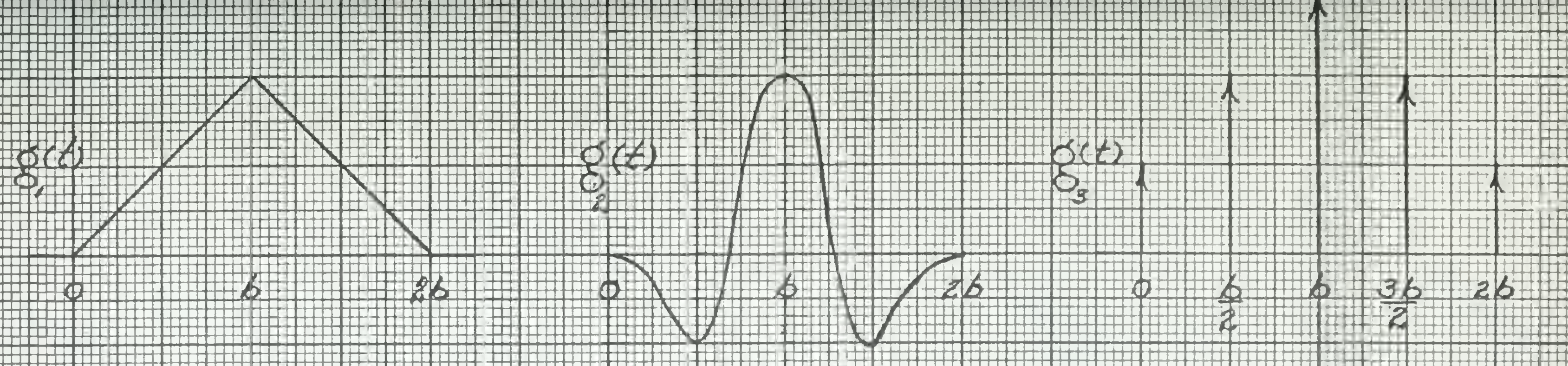


FIGURE 5

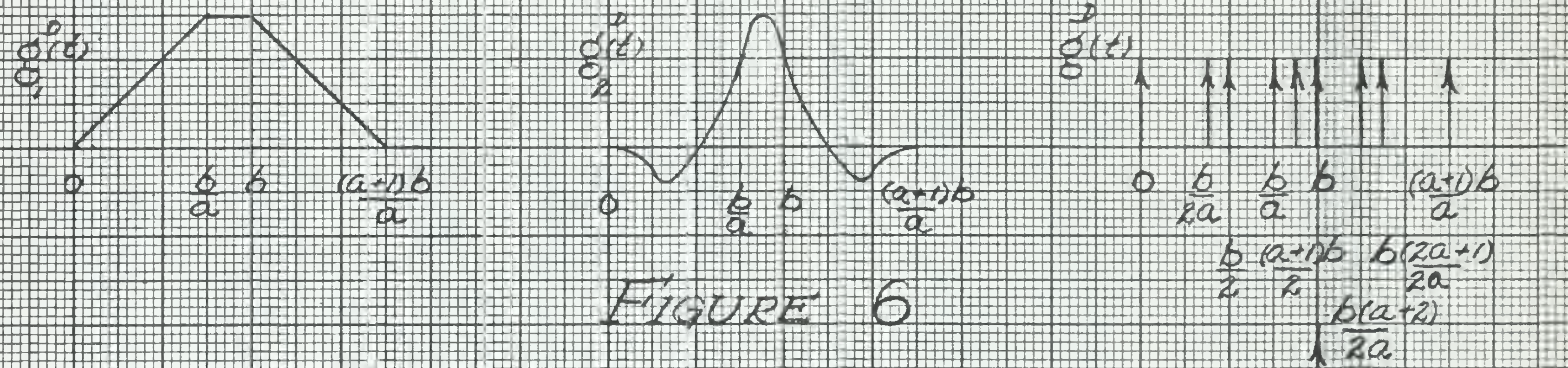


FIGURE 6

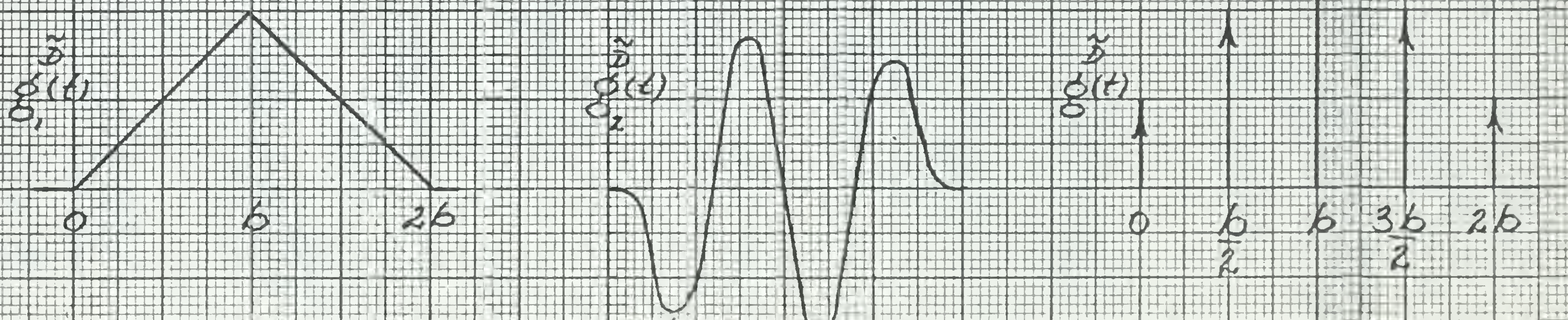


FIGURE 7

$$g_1^D(t) = h_1(t) * f_1^D(t) = \mathcal{L}^{-1} G_1^D(s) = \mathcal{L}^{-1} H_1(s) F^D(s)$$

$$= \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{e^{-bs}}{s}\right)\frac{1}{a}\left(\frac{a}{s} - \frac{a e^{-\frac{b}{a}s}}{s}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{1}{s^2} - \frac{e^{-\frac{b}{a}s}}{s^2} - \frac{e^{-bs}}{s^2} \quad \frac{e^{-(1+\frac{1}{a})bs}}{s^2}\right)$$

$$= t - \left(t - \frac{b}{a}\right) - (t-b) + \left(t - \frac{a+1}{a}b\right)$$

$$g_2^D(t) = h_2(t) * f_2^D(t) = \mathcal{L}^{-1} G_2^D(s) = \mathcal{L}^{-1} H_2(s) F_2^D(s)$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+\left(\frac{2\pi}{b}\right)^2} - \frac{e^{-bs}}{s^2+\left(\frac{2\pi}{b}\right)^2}\right\} a\left(\frac{2\pi}{b}\right)^2 \times$$

$$\left\{\frac{1}{s^2+\left(\frac{2\pi a}{b}\right)^2} - \frac{e^{-\frac{b}{a}s}}{s^2+\left(\frac{2\pi a}{b}\right)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{a}{\left[s^2+\left(\frac{2\pi}{b}\right)^2\right]\left[s^2+\left(\frac{2\pi a}{b}\right)^2\right]}\right.$$

$$- \frac{a\, e^{-(1+\frac{1}{a})bs}}{\left[s^2+\left(\frac{2\pi}{b}\right)^2\right]\left[s^2+\left(\frac{2\pi a}{b}\right)^2\right]}$$

$$- \frac{a\, e^{-\frac{b}{a}s}}{\left[s^2+\left(\frac{2\pi}{b}\right)^2\right]\left[s^2+\left(\frac{2\pi a}{b}\right)^2\right]}$$

$$\left. + \frac{a\, e^{-bs}}{\left[s^2+\left(\frac{2\pi}{b}\right)^2\right]\left[s^2+\left(\frac{2\pi a}{b}\right)^2\right]}\right\}\left(\frac{2\pi}{b}\right)^2$$

$$g_2^D(t) = \frac{\left(\frac{b}{2\pi}\right)\mathrm{SIN}\left(\frac{2\pi a}{b}\right)t - \left(\frac{ab}{2\pi}\right)\mathrm{SIN}\left(\frac{2\pi}{b}\right)t}{a^2-1}$$

$$0 < t < \frac{b}{a}$$

$$g_2^D(t) = -\frac{\left(\frac{ab}{2\pi}\right)\left\{\sin\left(\frac{2\pi}{b}\right)t - \sin\left(\frac{2\pi}{b}\right)\left(t-\frac{b}{a}\right)\right\}}{a^2-1} \qquad \frac{b}{a} < t < b$$

$$= \frac{\left(\frac{ab}{2\pi}\right)\sin\left(\frac{2\pi}{b}\right)\left(t-\frac{b}{a}\right) - \left(\frac{b}{2\pi}\right)\sin\left(\frac{2\pi a}{b}\right)(t-b)}{a^2-1} \qquad b < t < \left(\frac{1+a}{a}\right)b$$

$$= 0 \qquad \left(\frac{1+a}{a}\right)b < t$$

$$g_3^D(t) = h_3(t) * f_3^D(t) = \mathcal{L}^{-1}G_3^D(s) = \mathcal{L}^{-1}H_3(s)F_3^D(s)$$

$$= \mathcal{L}^{-1}\left\{1 + e^{-\frac{b}{2}s} + e^{-bs}\right\}\left\{\frac{1 + e^{-\frac{b}{2a}s} + e^{-\frac{b}{a}s}}{a}\right\}$$

$$= \frac{1}{a}\left\{\delta(t) + \delta\left(t-\frac{b}{2a}\right) + \delta\left(t-\frac{b}{2}\right)\right.$$
$$\delta\left(t-\frac{b}{a}\right) + \delta\left(t-\left[\frac{a+1}{2}\right]b\right) + \delta(t-b)$$
$$+\delta\left(t-\left[\frac{a+2}{2a}\right]b\right) + \delta\left(t-\left[\frac{2a+1}{2a}\right]b\right)$$
$$\left.+\delta\left(t-\left[\frac{a+1}{a}\right]b\right)\right\}$$

These three outputs are shown in Figure 6.

For purposes of comparison, the response of a matched filter to the usual approximation of the doppelered signal is computed for the three sample signals and shown in Figure 7.

$$g_1^{\tilde{D}}(t) = h_1(t) * f_1^{\tilde{D}}(t) = \mathcal{L}^{-1}G_1^{\tilde{D}}(t) = \mathcal{L}^{-1}H_1(s)F_1^{\tilde{D}}(s)$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{e^{-bs}}{s}\right\}^2$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{2e^{-bs}}{s^2} + \frac{e^{-2bs}}{s^2}\right\}$$

$$= t - 2(t-b) + (t-2b)$$

$$q_1^{\tilde{D}} = t \qquad 0<t<b$$
$$= (2b-t) \qquad b<t<2b$$
$$= 0 \qquad 2b<t$$

$$q_2^{\tilde{D}}(t) = h_2(t) * f_2^{\tilde{D}}(t) = \mathcal{L}^{-1} G_2^{\tilde{D}}(s) = \mathcal{L}^{-1} H_2(s) F_2^{\tilde{D}}(s)$$

$$= \mathcal{L}^{-1}\left\{ \frac{\frac{2\pi}{b}}{s^2+(\frac{2\pi}{b})^2} - \frac{\frac{2\pi}{b} e^{-bs}}{s^2+(\frac{2\pi}{b})^2} \right\} \times$$
$$\left\{ \frac{\frac{2\pi a}{b}}{s^2+(\frac{2\pi a}{b})^2} - \frac{\frac{2\pi a}{b} \cos 2\pi a\, e^{-bs}}{s^2+(\frac{2\pi a}{b})^2} - \frac{\sin 2\pi a\, e^{-bs} s}{s^2+(\frac{2\pi a}{b})^2} \right\}$$

$$= \frac{a \sin \frac{2\pi}{b} t - \sin \frac{2\pi a}{b} t}{\frac{2\pi}{b}(a^2-1)} \qquad 0<t<b$$

$$= \frac{\sin \frac{2\pi a}{b}(t-b) - \sin 2\pi a \cos \frac{2\pi}{b} t}{\frac{2\pi}{b}(a^2-1)}$$
$$- \frac{a \cos 2\pi a \sin \frac{2\pi}{b} t}{\frac{2\pi}{b}(a^2-1)} \qquad b<t<2b$$

$$= 0 \qquad 2b<t$$

$$q_3^{\tilde{D}}(t) = h_3(t) * f_3^{\tilde{D}}(t) = \mathcal{L}^{-1} G_3^{\tilde{D}}(s) = \mathcal{L}^{-1} H_3(s) F_3^{\tilde{D}}(s)$$
$$= \mathcal{L}^{-1}\left\{ 1 + e^{-\frac{b}{2}s} + e^{-bs} \right\}^2$$
$$= \delta(t) + 2\delta(t-\tfrac{b}{2}) + 3\delta(t-b) + 2\delta(t-\tfrac{3b}{2})$$
$$+ \delta(t-2b)$$

Some of the results of the previous derivations demonstrate interesting effects produced when an echo is received from a moving target. The time delay corresponding to travel time of the signal is $\frac{2R_o}{C+u}$ not $\frac{2R_o}{C}$ as is usually assumed. However this does not produce a range error if properly interpreted.

It is conventional to compute range r as $r = \frac{C\tau_o}{2}$ where as it has been shown $\tau_o = \frac{2R_o}{C+u}$

i.e.

$$r = \left(\frac{C}{2}\right)\frac{2R_o}{C+u} = \frac{CR_o}{C+u} = \frac{(C+u)R_o}{C+u} - \frac{uR_o}{C+u}$$
$$= R_o - \left(\frac{u}{2}\right)\frac{2R_o}{C+u} = R_o - \frac{u}{2}\tau_o$$

or the computed range is that of the target at a time $\frac{\tau_o}{2}$ before the echo is received. This value is known and can be introduced into the tracking or fire control problem to correct what small error would otherwise be present.

Other effects of doppler are a distortion of the expected output of a matched filter. These outputs, as can be seen from Figures 5 through 7 vary both in time and magnitude of maxima with resulting corrections to computed range and decision thresholds.

The previous examples have been chosen primarily to illustrate effects of doppler, rather than to illustrate any utility or complication which arises due to doppler effect. In the following section the pulse compression or "chirp" wave form is considered. It is one receiving considerable recent attention in radar and sonar and therefore is an interesting example. Further, it is desired to prove that a dopplered signal of this type contains information which may be advantageously retrieved.

The idea of the pulse compression signal is simple. The transmitted signal is an FM like signal whose frequency is a function of time, and a filter in the receiver whose delay is a function of frequency. A graphical explanation of the principle, which is somewhat in error, is given as introductory material, and to illustrate possible retrieval of information from the doppler effected signal other than range.

Assume a signal, as in Figure 8 which is periodically shifted in frequency. This signal is impressed on a filter-delay line having the characteristic shown in Figure 9. The dotted curves are a pulse approximation to the actual curve (solid line) where the pulses could be made very narrow. Thus the signal may be shown in Figure 10. The processing of the signal is as shown in Figure 11.

It is first assumed that the target is not moving and therefore is only a constant delay to the signal ($\tilde{\tau}_0$). The output of the signal from the filter, where the characteristic of each is approximated by the dotted rectangles in Figures 8 and 9, is shown in the time domain by Figure 12. $K = \tau' + 3\tau$ is a function of the filter and is a constant which can be subtracted from τ, to give $\tilde{\tau}_0$ from which target range can be computed. Note the signal processing gain which is an important property of the pulse compression signal.

Now assume the target is moving at a rate such as, by the doppler approximation, to give a frequency shift equal to $\Delta\omega$, and again also introduces a delay of $\tilde{\tau}_0$. The result of the processing of this signal is depicted in Figure 13.

It is seen now that using $K = \tau' + 3\tau$, the known delay of the filter and subtracting from the time of output $\tilde{\tau}_0 + \tau' + 2\tau$ gives

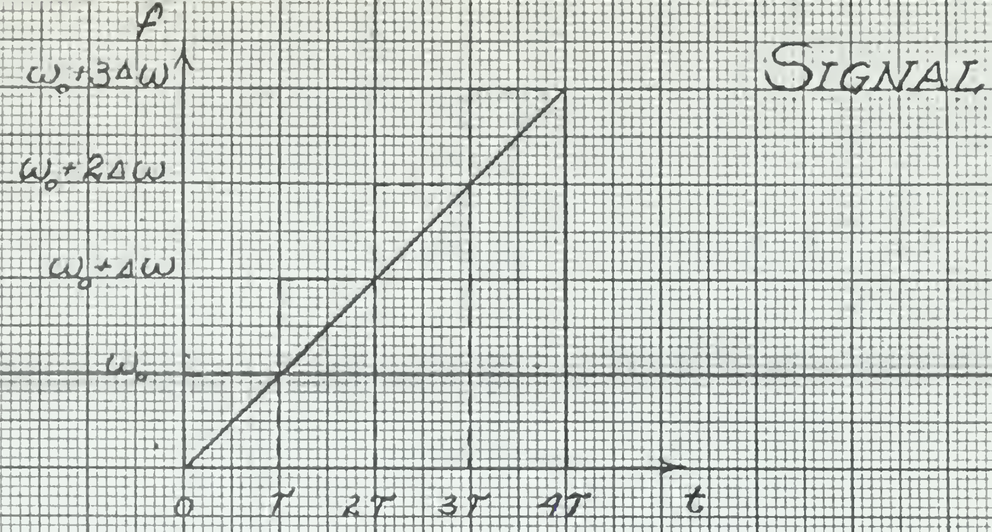


FIGURE 8

FIGURE 9

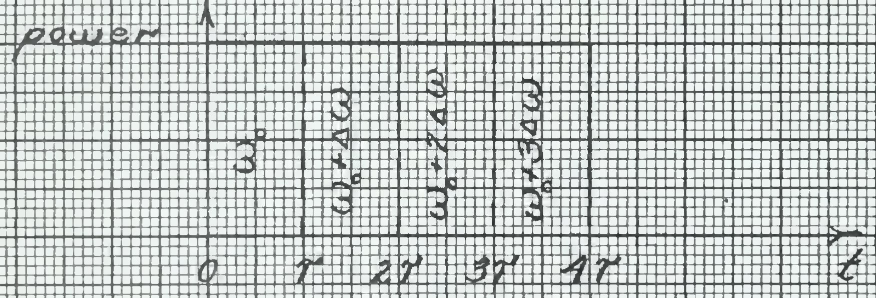


FIGURE 10

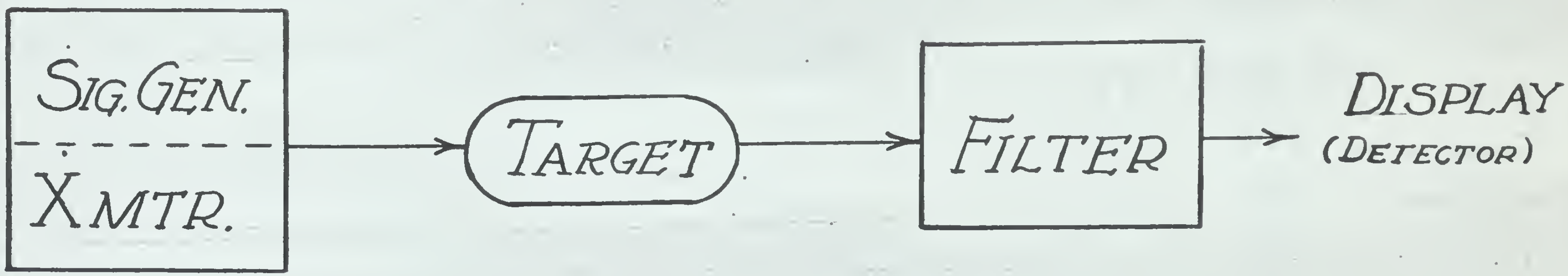


FIGURE 11

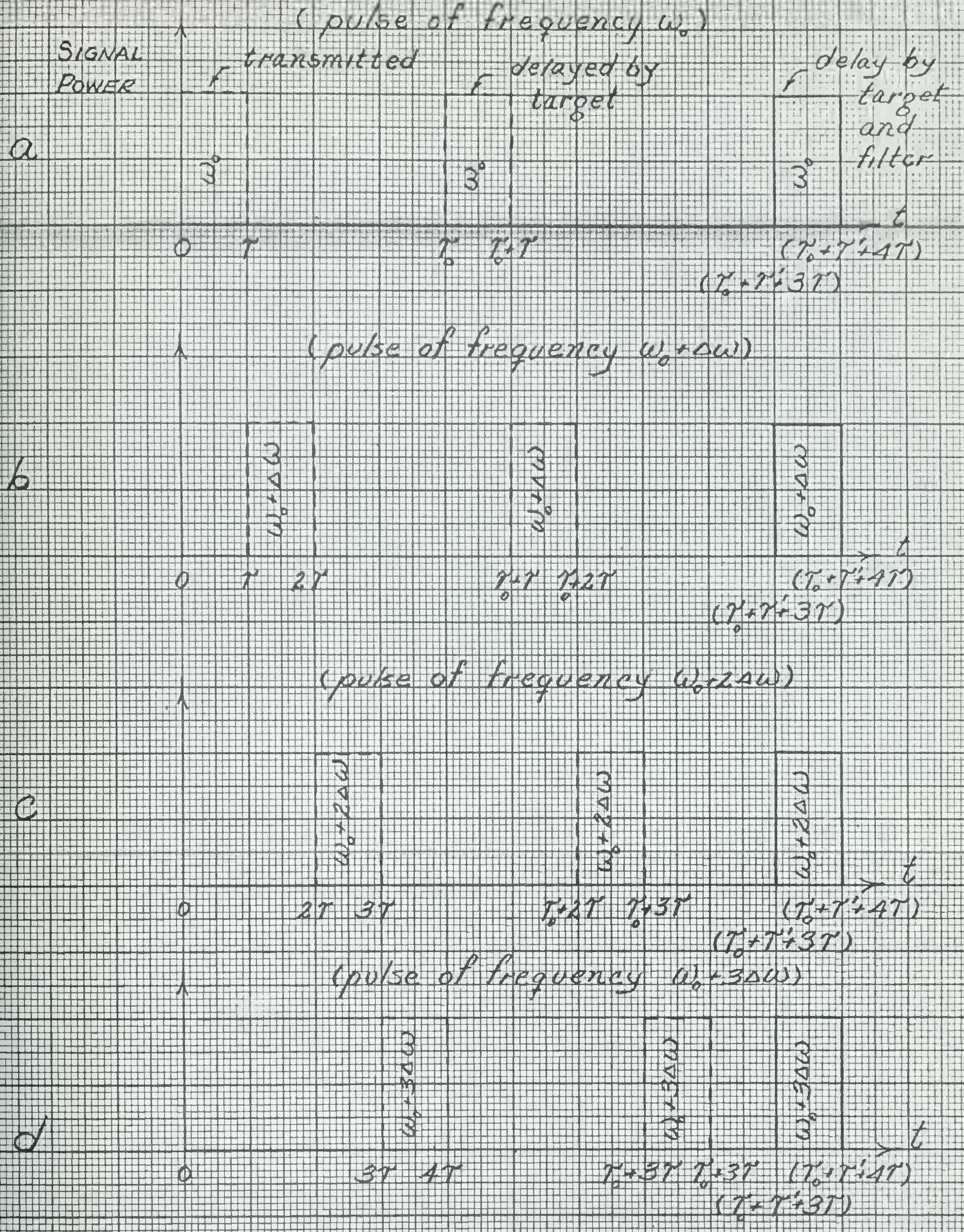

SIGNAL POWER
(pulse of frequency ω₀)
transmitted
delayed by target
delay by target and filter
a
b
c
d
(pulse of frequency ω₀+Δω)
(pulse of frequency ω₀+2Δω)
(pulse of frequency ω₀+3Δω)
t

FIGURE 12

e

(resultant of figs. (a), (b), (c), and (d))

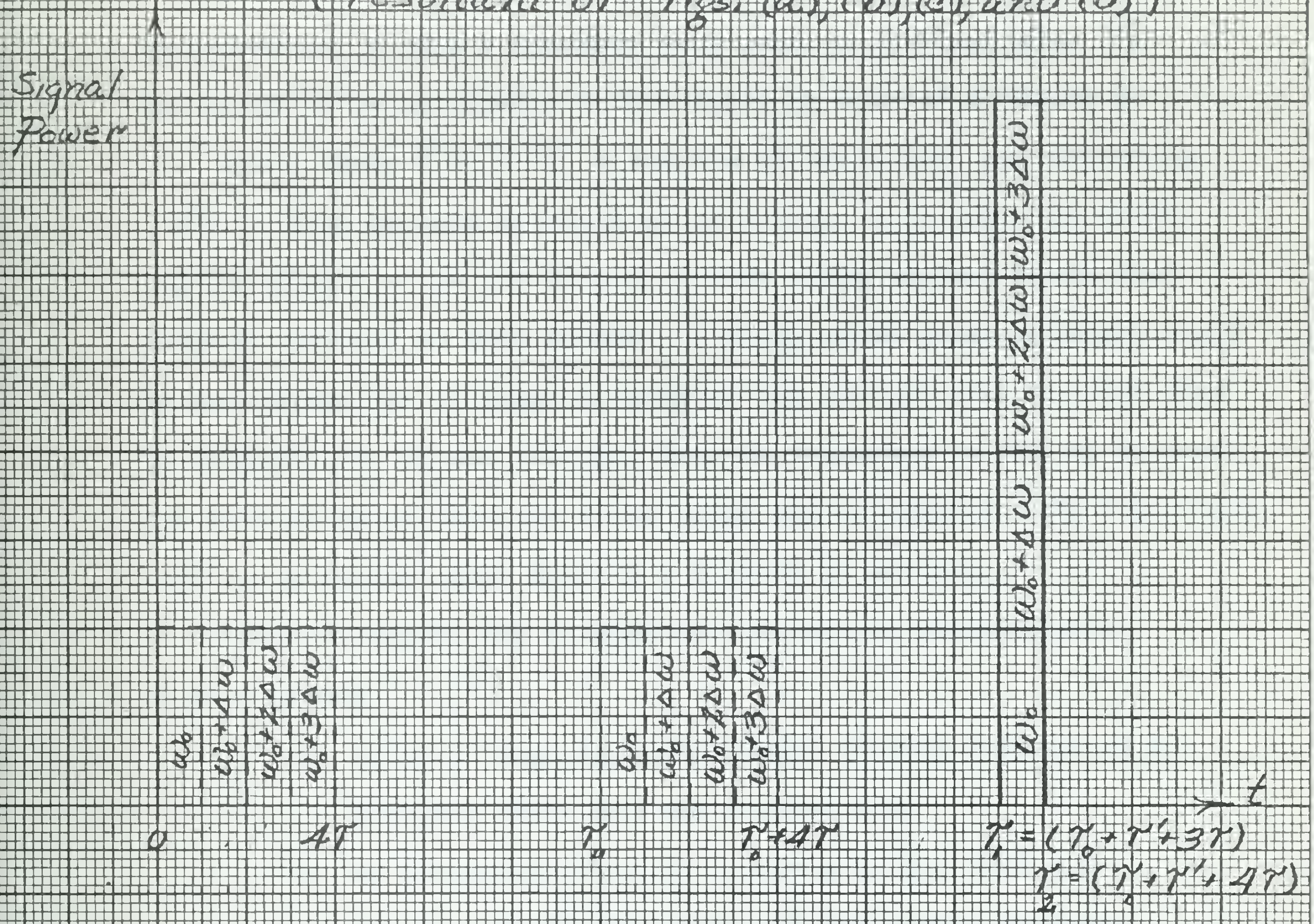


FIGURE 12 (cond)

$\tau_0 - \tau$. When target range is computed there is a resultant error due to the factor $-\tau$. Target range will be computed correctly from τ_0 only.

Assume now another signal and filter which may be described by Figures 14, 15, and 16 similar to Figures 8, 9, and 10.

Again assume the target is moving at a rate such as, in the doppler approximation, to give a frequency shift equal to $\Delta\omega$ and delay τ_0.

The result of the processing of this signal is depicted in Figure 17.

Again, using $K = \tau + 3\tau'$, a function of the filter, and subtracting K from the time of output of the filter, it is seen that comparing the outputs of the two systems, a fixed target appears at the same range as computed for both systems. A moving target gives erroneous ranges in each system, but if range is computed from the mean time accurate range is obtained. It is seen also that from the difference in time as obtained for each system the velocity of the target may be computed. The time shifts were due to doppler shift which is a function of velocity.

These two signals may be mixed and transmitted simultaneously. If this is done:

1. Accurate range and instantaneous velocity information are available after only one received echo.
2. As the probabilities of receiving an echo from each channel are not independent but are closely dependent, a decision criterian which requires an output of both channels to classify an output as echo would not increase the "missed target" probability by any appreciable amount.
3. The probabilities of an output from each channel due to

noise are relatively independent, and the false target probability of a criterian requiring two outputs each exceeding a threshold would be considerably reduced.

The ideas discussed in Section III are presented analytically in Section IV.

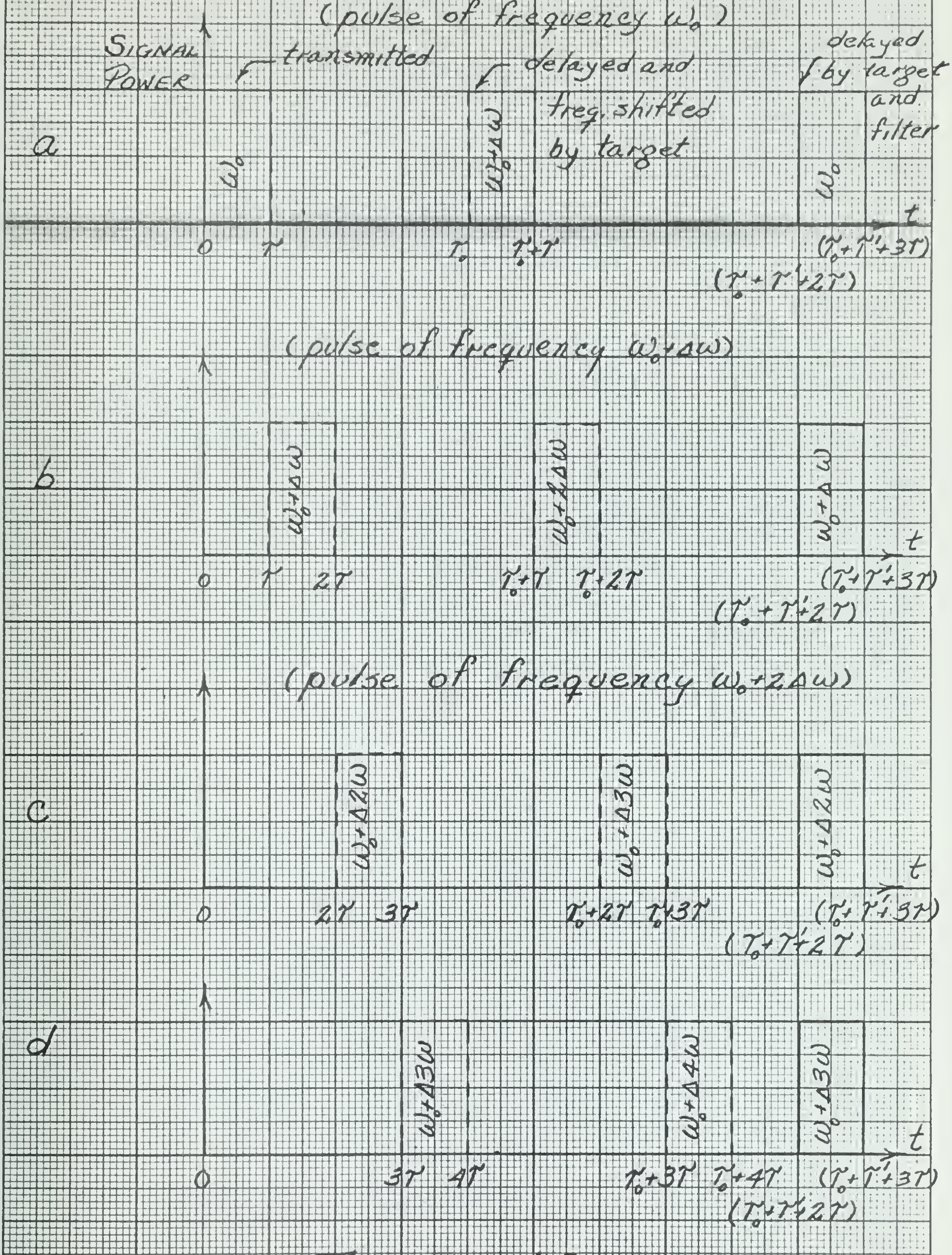

(pulse of frequency ω_0)
SIGNAL POWER
transmitted
delayed and freq. shifted by target
delayed by target and filter
a
ω_0
$\omega_0+\Delta\omega$
ω_0
t
0
τ
τ_0
$\tau_0'+\tau$
$(\tau_0+\tau'+3\tau)$
$(\tau_0'+\tau'+2\tau)$
(pulse of frequency $\omega_0+\Delta\omega$)
b
$\omega_0+\Delta\omega$
$\omega_0+2\Delta\omega$
$\omega_0+\Delta\omega$
t
0
τ
2τ
$\tau_0'+\tau$
$\tau_0'+2\tau$
$(\tau_0'+\tau'+3\tau)$
$(\tau_0'+\tau'+2\tau)$
(pulse of frequency $\omega_0+2\Delta\omega$)
c
$\omega_0+\Delta 2\omega$
$\omega_0+\Delta 3\omega$
$\omega_0+\Delta 2\omega$
t
0
2τ
3τ
$\tau_0+2\tau$
$\tau_0+3\tau$
$(\tau_0'+\tau'+3\tau)$
$(\tau_0+\tau'+2\tau)$
d
$\omega_0+\Delta 3\omega$
$\omega_0+\Delta 4\omega$
$\omega_0+\Delta 3\omega$
t
0
3τ
4τ
$\tau_0+3\tau$
$\tau_0+4\tau$
$(\tau_0+\tau'+3\tau)$
$(\tau_0+\tau'+2\tau)$

FIGURE 13

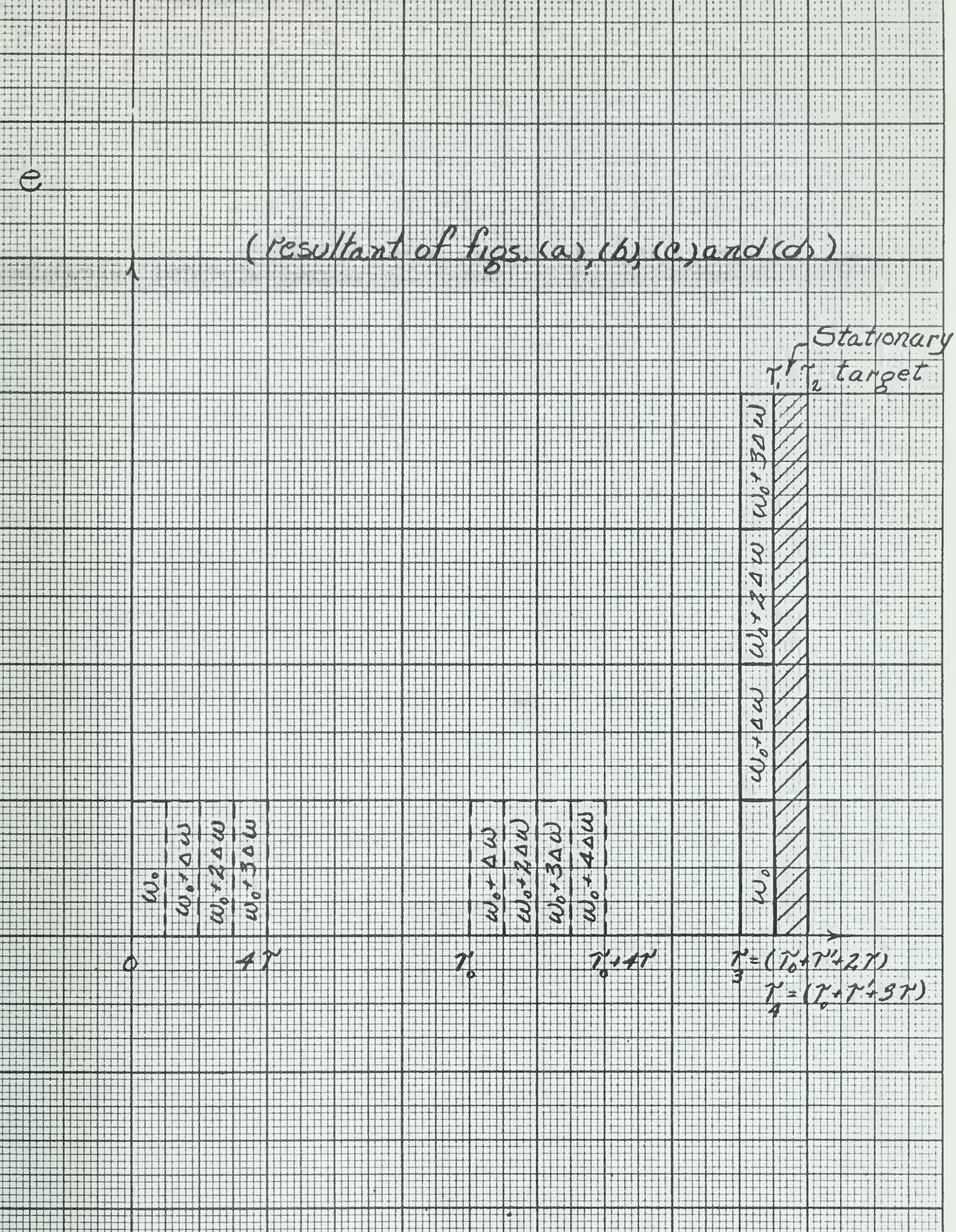


FIGURE 13 (cond)

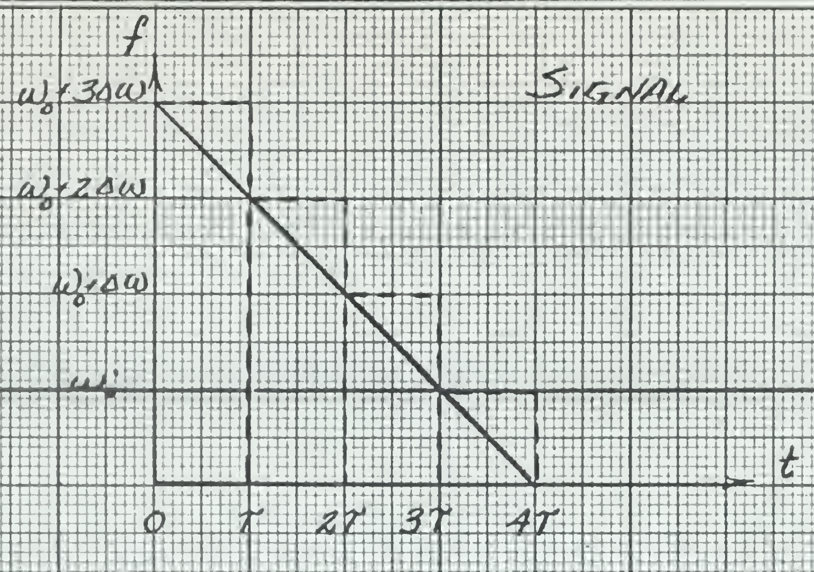


FIGURE 14

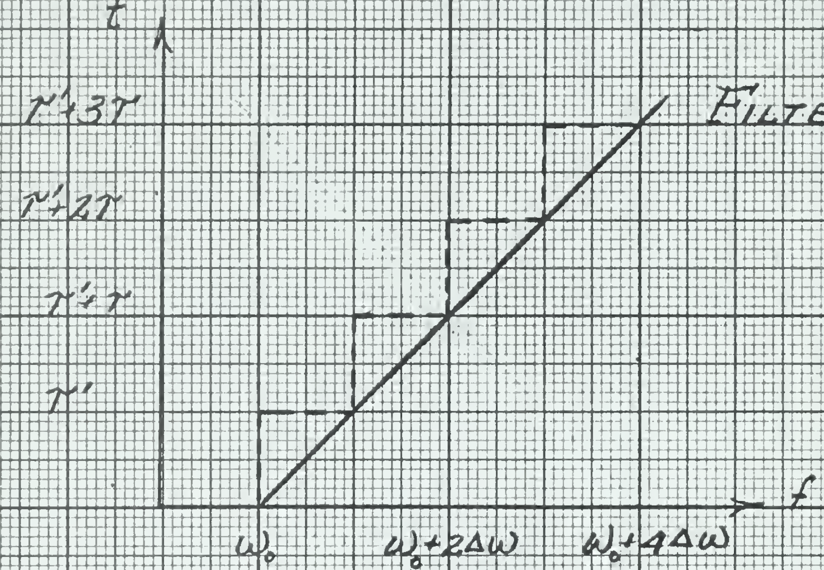


FIGURE 15

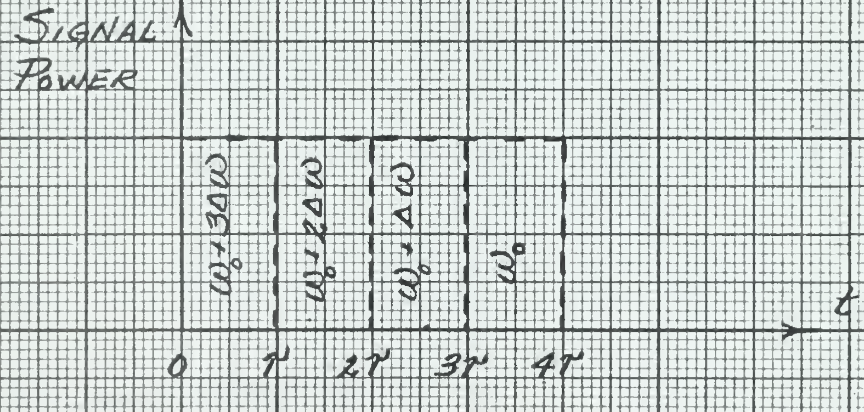


FIGURE 16

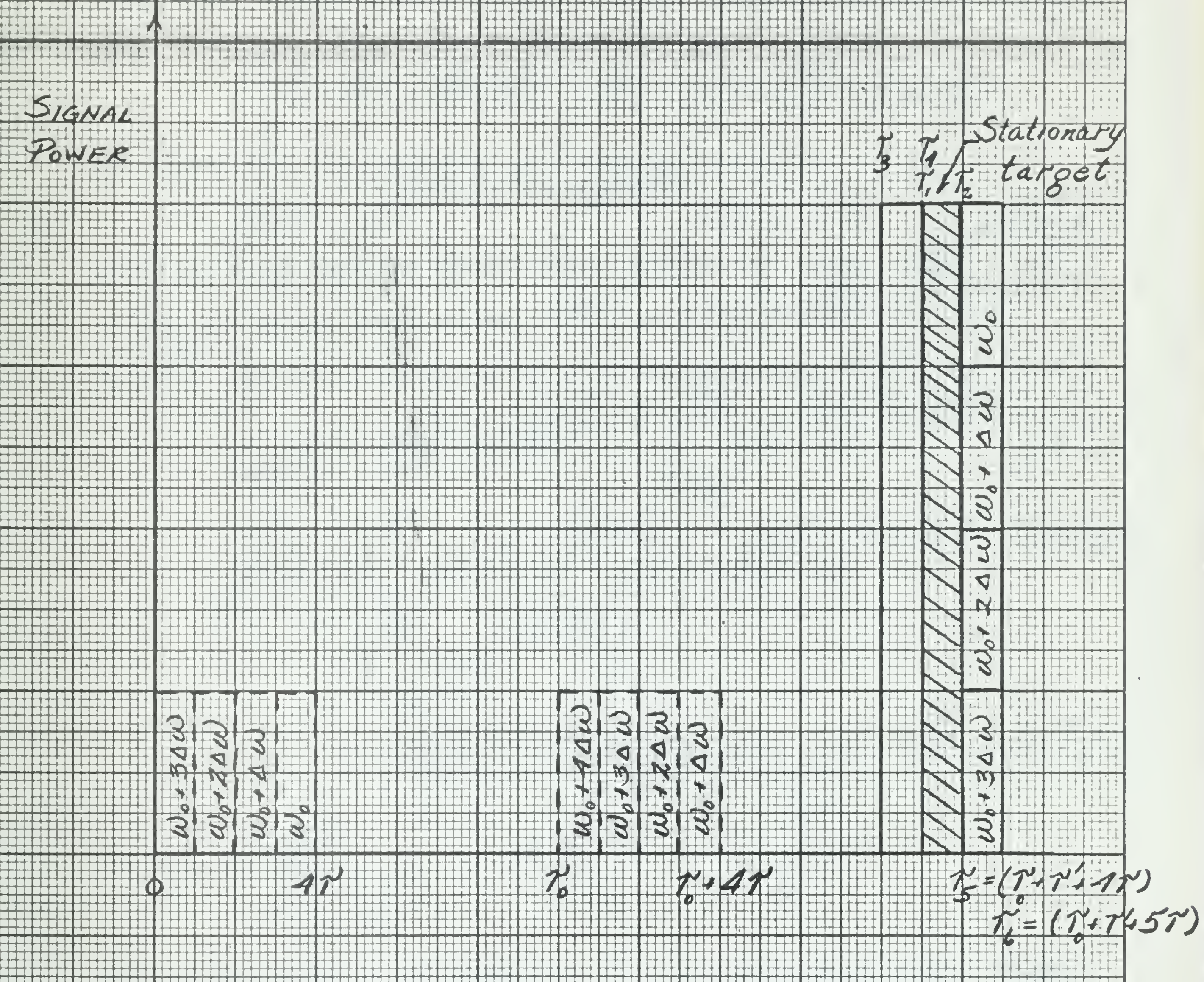

SIGNAL
POWER
Stationary
target
$\omega_0+3\Delta\omega$
$\omega_0+2\Delta\omega$
$\omega_0+\Delta\omega$
ω_0
$\omega_0+4\Delta\omega$
0
4τ
τ_0
$\tau_0+4\tau$

FIGURE 17

Various papers by Cook[1] and Klauder[2] have discussed in some detail the pulse compression signal. Therefore only a cursory treatment is given here. A signal whose frequency varies linearly with time is expressed as

$$f = f_0 + kt = \frac{1}{2\pi}\dot{\theta} \quad , \quad \theta = 2\pi\left(f_0 t + \tfrac{1}{2}kt^2\right)$$

The signal $f(t)$ is therefore, in exponential notation:

$$f(t) = e^{j2\pi\left(f_0 t + \frac{1}{2}kt^2\right)} \qquad -T/2 < t < T/2$$

Chin[3] has shown that the desired phase shift of the filter is $\theta = \dfrac{\pi(f_0 - f)^2}{k}$ or that the filter characteristic $H(f)$ is $H(f) = \sqrt{\dfrac{2k}{\pi}}\, e^{j\frac{\pi}{k}(f_0-f)^2}$

$G(f)$ the filter output spectrum is the product of $H(f)$ and $F(f)$

$$G(f) = \sqrt{\frac{2k}{\pi}}\, e^{j\frac{\pi}{k}(f_0-f)^2} \int_{-T/2}^{T/2} e^{j2\pi\left[\left(f_0 t + \frac{1}{2}kt^2\right) - ft\right]}\, dt$$

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df = \sqrt{\frac{2k}{\pi}}\, e^{j2\pi\left(f_0 t - \frac{1}{2}kt^2 + \frac{1}{8}\right)} \times \int_{-T/2}^{T/2} e^{j2\pi k t \tau}\, d\tau$$

$$= \sqrt{\frac{2kT^2}{\pi}} \left(\frac{\mathrm{SIN}(\pi k T t)}{\pi k T t}\right) e^{j2\pi\left(f_0 t - \frac{1}{2}kt^2 + \frac{1}{8}\right)}$$

The gain is $\dfrac{P_{max\,out}}{P_{max\,in}} = \dfrac{\frac{2kT^2}{\pi}}{1} = \dfrac{2kT^2}{\pi}$

and the resolution is increased from T, the input pulse width, to $\dfrac{2}{kT}$.

[1] C. E. Cook, Pulse Compression--Key to More Efficient Radar Transmission, pp. 310-316, IRE Proc. Mar. 1960.

[2] J. R. Klauder, et al, The Theory and Design of Chirp Radars, pp. 745-808, B.S.Y.J., July, 1960.

[3] J. E. Chin and C. E. Cook, The Mathematics of Pulse Compression-A Problem in Systems Analysis, Sperry Eng. Review, pp. 11-16, Oct. 1959.

The filter proposed by Chin is not a matched filter and the S/N ratio gain of such a filter is not a maximum. Klauder[4] has evaluated $g(t)$ for the case in which $H(f)$ is a matched filter. The envelope of the response so obtained is:

$$g(t) = T\,\frac{\sin \pi(kT|t| - kt^2)}{\pi kT|t|} \qquad |t| < T$$

$$= 0 \qquad \text{elsewhere}$$

[4] J. R. Klauder, et al, The Theory and Design of Chirp Radars, pp. 745-808, July, 1960.

When the doppler effected signal may be approximated by the usual approximation, i e., shifting the entire spectrum by a uniform amount, the heuroisticly predicted results are easily proven analytically.

Given:

$$f = f_0 + kt$$

$$f_2(t) = e^{j2\pi(f_0 t + \frac{1}{2}kt^2)}$$

It is known that the shift due to doppler effect is:

$$\Delta f = (a-1) f_0$$

$$f_D = f_0 + \Delta f = a f_0$$

Therefore:

$$f_2^{\tilde{D}}(t) = e^{j2\pi(f_D t + \frac{1}{2}kt^2)}$$

$$h_2(t) = e^{j2\pi(f_0 t - \frac{1}{2}kt^2)}$$

$$g_2^{\tilde{D}}(t) = f_2^{\tilde{D}}(t) * h_2(t)$$

$$g_2^{\tilde{D}}(t) = \int_{-T/2}^{T/2} e^{j2\pi(f_D\tau + \frac{1}{2}k\tau^2)} e^{j2\pi(f_0[t-\tau] - \frac{1}{2}k[t-\tau]^2)} d\tau$$

$$= e^{j2\pi(f_D t - \frac{1}{2}kt^2)} \int_{-T/2}^{T/2} e^{j2\pi k(t + \frac{\Delta f}{k})\tau} d\tau$$

$$= T e^{j2\pi(f_D - \frac{1}{2}kt^2)} \left\{ \frac{\sin \pi k T(t + \frac{\Delta f}{k})}{\pi k T(t + \frac{\Delta f}{k})} \right\}$$

Or as predicted the envelope of the output is shifted in time and the amount is:

$$\tau = \frac{\Delta f}{k} = \frac{(a-1) f_o}{k} = \frac{2u}{(c-u)} \times \frac{f_o}{k} \cong \frac{2u}{c} \times \frac{f_o}{k}$$

$$u = \frac{\tau c k}{2 f_o}$$

It may be similarly shown that the companion signal would be shifted by an amount $-\tau$.

If a matched filter:

$$h_2(t) = e^{j2\pi(f_o t - \frac{1}{2}kt^2)} \qquad |t| < T/2$$

as discussed by Klauder is considered, the envelope of the response $g_2^{\tilde{D}}(t)$ is found to be:

$$g_2^{\tilde{D}}(t) = T \frac{\text{SIN}\ \pi k \{|t + \frac{\Delta f}{k}|(T + \frac{\Delta f}{k}) + (t + \frac{\Delta f}{k})^2\}}{\pi k |t + \frac{\Delta f}{k}|}$$

$$= 0 \qquad |t| > T$$

However, if the doppler approximation was reasonable:

$$\Delta f << Tk \quad \text{or} \quad \frac{\Delta f}{k} << T$$

Therefore:

$$g_2^{\tilde{D}}(t) \cong T \frac{\text{SIN}\ \pi k \{|t + \frac{\Delta f}{k}|T + (t + \frac{\Delta f}{k})^2\}}{\pi k T |t + \frac{\Delta f}{k}|}$$

$$= 0 \qquad |t| > T$$

and it is again seen that this envelope is shifted by an amount $\frac{\Delta f}{k}$.

The general case is not so easily seen.

$$f_1(t) = \text{RECT}(\tfrac{t}{T})\, e^{j2\pi(f_o t - \frac{k}{2}t^2)}$$

$$h_1(t) = \text{RECT}(\tfrac{t}{T})\, e^{j2\pi(f_o t + \frac{k}{2}t^2)}$$

$$f_1^D(t) = \mathrm{RECT}\left(\frac{at}{T}\right) e^{j2\pi(f_0 a t - \frac{k}{2} a^2 t^2)}$$

$$g_1^D(t) = f_1^D(t) * h_1(t)$$

$$= \int_{-\infty}^{\infty} \mathrm{RECT}\left(\frac{a\tau}{T}\right) e^{j2\pi(f_0 a \tau - \frac{k}{2}\tau^2)} \mathrm{RECT}\left(\frac{t-\tau}{T}\right) \times e^{j2\pi(f_0[t-\tau] + \frac{k}{2}[t-\tau]^2)} \, d\tau$$

$$g_1^D(t) = x_1^D(t) + y_1^D(t)$$

$$x_1^D(t) = \begin{cases} \displaystyle\int_{-T/2a}^{T/2a} f^D(\tau)\, h_1(t-\tau)\, d\tau & 0 \le t \le \frac{T}{2}\frac{(a-1)}{a} \\ 0 & \text{elsewhere} \end{cases}$$

$$y_1^D(t) = \begin{cases} \displaystyle\int_{t-T/2}^{T/2a} f_1^D(t)\, h_1(t-\tau)\, d\tau & \frac{T}{2}\frac{(a-1)}{a} \le t \le \frac{T}{2}\frac{(a+1)}{a} \\ 0 & \text{elsewhere} \end{cases}$$

$$x_1^D(t) = e^{j2\pi\left\{f_0 t + \frac{1}{2}kt^2 + \frac{(f_0[a-1]-kt)^2}{2k[a^2-1]}\right\}} \times \frac{1}{\sqrt{2k(a^2-1)}}\left[Z^*(u_{12}) - Z^*(u_{11})\right] \quad 0 \le t \le \frac{T}{2}\frac{(a-1)}{a}$$

$$y_1^D(t) = e^{j2\pi\left\{f_0 t + \frac{1}{2}kt^2 + \frac{(f_0[a-1]-kt)^2}{2k[a^2-1]}\right\}} \times \frac{1}{\sqrt{2k(a^2-1)}}\left[Z^*(u_{12}) - Z^*(u_{10})\right]$$
$$\frac{T}{2}\frac{(a-1)}{a} \le t \le \frac{T}{2}\frac{(a+1)}{a}$$

Where:

$$u_{11} = \sqrt{2k(a^2-1)}\left[-T/2a - \left\{\frac{\left(t - \frac{f_0[a-1]}{k}\right)}{(a^2-1)}\right\}\right]$$

$$u_{12} = \sqrt{2k(a^2-1)}\left[T/2a - \left\{\frac{\left(t - \frac{f_0[a-1]}{k}\right)}{(a^2-1)}\right\}\right]$$

$$u_{10} = \sqrt{2k(a^2-1)}\left[(t - T/2a) - \left\{\frac{\left(t - \frac{f_0[a-1]}{k}\right)}{(a^2-1)}\right\}\right]$$

$$= \sqrt{2k(a^2-1)}\left[-T/2a - \left\{\frac{\left(t[2-a^2] - \frac{f_0[a-1]}{k}\right)}{(a^2-1)}\right\}\right]$$

$$\cong \sqrt{2k(a^2-1)}\left[-T/2a - \left\{\frac{\left(t - \frac{f_0[a-1]}{k}\right)}{(a^2-1)}\right\}\right] \quad \text{if } a^2 \cong 1$$

The companion signal:

$$f_2^D(t) = RECT\left(\frac{at}{T}\right) e^{j2\pi(f_0 at + \frac{1}{2}ka^2t^2)}$$

convoluted with the matched filter:

$$h_2(t) = RECT\left(\frac{t}{T}\right) e^{j2\pi(f_0 t - \frac{1}{2}kt^2)}$$

yields:

$$g_2^D(t) = f_2^D(t) * h_2(t) = x_2^D(t) + y_2^D(t)$$

$$x_2^D(t) = e^{j2\pi\left\{f_0 t - \frac{1}{2}kt^2 - \frac{(f_0[a-1]+kt)^2}{k(a^2-1)}\right\}} \times \frac{1}{\sqrt{2k(a^2-1)}}\left[Z^*(u_{22}) - Z^*(u_{21})\right]$$

$$0 \le t \le \frac{T}{2}\frac{(a-1)}{a}$$

$$y_2^D(t) = e^{j2\pi\left\{f_0 t - \frac{1}{2}kt^2 - \frac{(f_0[a-1]+kt)^2}{k(a^2-1)}\right\}} \times \frac{1}{\sqrt{2k(a^2-1)}}\left[Z^*(u_{22}) - Z^*(u_{20})\right]$$

$$\frac{T}{2}\frac{(a-1)}{a} \le t \le \frac{T}{2}\frac{(a+1)}{a}$$

$$u_{21} = \sqrt{2k(a^2-1)}\left[-\frac{T}{2a} - \left\{\frac{\left(t + \frac{f_0[a-1]}{k}\right)}{(a^2-1)}\right\}\right]$$

$$u_{22} = \sqrt{2k(a^2-1)}\left[\frac{T}{2a} - \left\{\frac{\left(t + \frac{f_0[a-1]}{k}\right)}{(a^2-1)}\right\}\right]$$

$$u_{20} = \sqrt{2k(a^2-1)}\left[-\frac{T}{2a} - \left\{\frac{\left(t + \frac{f_0[a-1]}{k}\right)}{(a^2-1)}\right\}\right] \quad \text{if } a^2 \cong 1$$

The envelope of $g_1^D(t)$ is seen to be:

$$\epsilon_1^D(t) = \begin{cases} Z(u_{12}) - Z(u_{11}) & 0 \le t \le \frac{T}{2}\frac{(a-1)}{a} \\ Z(u_{12}) - Z(u_{10}) & \frac{T}{2}\frac{(a-1)}{a} \le t \le \frac{T}{2}\frac{(a+1)}{a} \end{cases}$$

and the envelope of $g_2^D(t)$ is:

$$\epsilon_2^D(t) = \begin{cases} Z(u_{22}) - Z(u_{21}) & 0 \le t \le \frac{T}{2}\frac{(a-1)}{a} \\ Z(u_{22}) - Z(u_{10}) & \frac{T}{2}\frac{(a-1)}{a} \le t \le \frac{T}{2}\frac{(a+1)}{a} \end{cases}$$

where it is found throughout that:

$$u_2(t) = u_1\left(t + \frac{2f_0[a-1]}{k}\right)$$

Therefore it is seen that there is a time difference between the two signals

$$2\tau = \frac{2f_0[a-1]}{k} = \frac{2\Delta f}{k}$$

which is the same term found by employing the doppler approximation.

CONCLUSION:

A system which obtains simultaneous target range and radial velocity without tracking has been proposed. The pair of pulse compression signals in the proposed system have a mean output time of $\tau_o = \frac{2R_o}{c+u}$ such that $r = \frac{\tau_o}{2c}$ is the correct range of the target at a time $\frac{\tau_o}{2}$ prior to the reception of the signal. The difference between the output times of the pair of signals is $2\tau = \frac{2f_o(a-1)}{k}$ such that $u = \frac{\tau c k}{(2f_o - \tau k)}$. Accurate determination of this velocity is limited by the resolution of the signals in time. The resolution of the system in the absence of the noise is $\frac{2}{kT}$ which limits the velocity resolution of the system to $\Delta u = \frac{c}{2f_o kT}$

Of interest for further study is the resolution of the system and the false and missed target probabilities in the presence of noise. The design criteria for the various parameters k, T and f_o have not been studied and the performance of the system proposed depends on the interrelation of these parameters. If the above studies continue to show the system to be promising it is hoped that laboratory tests of the system will be made.

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